Linear Momentum

* $\vec{P} = m\vec{v}$ (linear momentum of a particle)

* If the particle is at Rest $\Rightarrow \vec{v} = 0 \Rightarrow \vec{P} = 0$

* $\vec{v}$ is antiderivative of $\vec{a}$ $\quad \vec{v} = \int \vec{a}\,dt$

* Position vector: $\vec{r}_{(t)} = \int \vec{v}\,dt$

* Particle falls freely $\Rightarrow \vec{a} = \vec{g} \Rightarrow a = g\vec{j}$

* Position vector of center of mass:

$$\vec{OG} = \vec{r}_G = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 \ldots}{m_1 + m_2 + m_3 \ldots}$$

* linear momentum of system: $\vec{P}_{system} = \vec{P}_1 + \vec{P}_2 + \vec{P}_n \cdots = m_1\vec{v}_1 + m_2\vec{v}_2 + m_n\vec{v}_n$

* linear momentum $\vec{P}_G$ of center of mass: $M\vec{V}_G = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n$

$$\Rightarrow \vec{P}_G = M\vec{V}_G = \vec{P}_{system}$$

* Newton's 2nd law:

$$\boxed{\sum \vec{F}_{ext} = \frac{d\vec{P}}{dt}}$$

* Theorem of center of mass:

$$\vec{P}_{system} = \vec{P}_G \Rightarrow \boxed{\sum \vec{F}_{ext\,system} = \frac{d\vec{P}_G}{dt}}$$

$$\Rightarrow \boxed{(\sum \vec{F}_{ext})_{system} = M\vec{a}_G}$$

* Conservation of linear momentum:
  * system is isolated $\Rightarrow (\sum \vec{F}_{ext})_{system} = \vec{0}$

    $\Rightarrow \vec{P}_{system}$ is conserved

    $\Rightarrow (\vec{P}_{system})_{before} = (\vec{P}_{system})_{after}$

    $\Rightarrow (\vec{P}_A + \vec{P}_B)_b = (\vec{P}'_A + \vec{P}'_B)_{after}$
  * $\vec{P}_{system}$ is conserved $\Rightarrow \vec{P}_G$ is conserved $\Rightarrow \vec{P}_G$ is constant $\Rightarrow$ 0: G is at rest; $\neq 0$: G is in U.R.M

* Types of collision:

(1) Elastic: $\begin{cases} (\vec{P}_{system})_{j.before} = (\vec{P}_{system})_{just\ after} \\ (K.E)_{system\ before} = (K.E)_{system\ j.a.} \end{cases} \Rightarrow \begin{cases} \vec{P}_1 + \vec{P}_2 = \vec{P}'_1 + \vec{P}'_2 \\ K.E_1 + K.E_2 = K.E'_1 + K.E'_2 \end{cases}$

② Non-Elastic: $\begin{cases} \vec{P}_{j.b} = \vec{P}_{j.a} \\ (K.E)_{j.b} > (K.E)_{j.a} \end{cases}$

$$K.E_{lost} = K.E_{j.b} - K.E_{j.a}$$

* Head-on collision (velocities are collinear):

$\begin{cases} (\vec{P}_{sys})_{j.b} = (\vec{P}_{sys})_{j.a} \\ (K.E)_{j.b} = (K.E)_{sys\ j.a} \end{cases} \Rightarrow \begin{cases} m_A\vec{V}_A + m_B\vec{V}_B = m_A\vec{V}'_A + m_B\vec{V}'_B \\ \frac{1}{2}m_A\vec{V}_A^2 + \frac{1}{2}m_B\vec{V}_B^2 = \frac{1}{2}m_A\vec{V}_A'^2 + \frac{1}{2}m_B\vec{V}_B'^2 \end{cases}$

Since velocities are collinear $\Rightarrow$ vectors are replaced by corresponding algebric values

$\Rightarrow \begin{cases} m_AV_A + m_BV_B = m_AV'_A + m_BV'_B \text{ ...eq①} \\ m_AV_A^2 + m_BV_B^2 = m_AV_A'^2 + m_BV_B'^2 \end{cases} \Rightarrow \begin{cases} m_A(V_A - V_A') = m_B(V_B' - V_B) \\ m_A(V_A^2 + V_A'^2) = m_B(V_B'^2 - V_B^2) \end{cases}$

$\Rightarrow \begin{cases} m_A(V_A - V_A') = m_B(V_B' - V_B) \text{ ... eq②} \\ m_A(V_A - V_A')(V_A + V_A') = m_B(V_B' - V_B)(V_B' + V_B) \text{ -- eq③} \end{cases}$

$\frac{eq②}{eq③}$: $\Rightarrow V_A + V_A' = V_B' + V_B$ -- eq④ $\Rightarrow \begin{cases} m_AV_A + m_BV_B = m_AV_A' + m_BV_B' \text{ --eq①} \\ V_A + V_A' = V_B' + V_B \text{ -- eq④} \end{cases}$

$\Rightarrow \begin{cases} m_AV_A' + m_BV_B' = m_AV_A + m_BV_B \text{ -- ①} \\ V_A' - V_B' = V_B - V_A \text{ --④} \end{cases}$

$\Rightarrow V_B' = V'_A - V_B + V_A$

sub in eq①

$m_AV_A' + m_B(V_A' - V_B + V_A) = m_AV_A + m_BV_B$

$m_AV_A' + m_BV'_A - m_BV_B + m_BV_A = m_AV_A + m_BV_B$

$\Rightarrow (m_A + m_B)V'_A = (m_A - m_B)V_A + 2m_BV_B$

$$\Rightarrow \boxed{V'_A = \frac{m_A - m_B}{m_A + m_B}V_A + \frac{2m_B}{m_A + m_B}V_B}$$

sub V'_A in eq④

$$\Rightarrow \boxed{V'_B = \frac{2m_A}{m_A + m_B}V_A + \frac{m_B - m_A}{m_A + m_B}V_B}$$

Oscillations

- Period : $\boxed{T = \frac{1}{F}}$ $\boxed{T = \frac{t}{n}}$ $\boxed{F = \frac{n}{t}}$
- Frequency : $\boxed{F = \frac{1}{T}}$

* Free oscillations:
  - damped (Pseudo periodic motion): -oscillates under the effect of dissipative force (Friction)
    - amplitude and mechanical energy are not conserved.
    - period is called pseudo period T $(T > T_0)$
  - nondamped (Simple Harmonic motion S.H.M): -oscillates without friction or any dissipative force.
    - Amplitude and mechanical energy are conserved.
    - period is called proper period (Natural) T_0
    - T_0 : is the smallest possible period of oscillations.

* Expression of M.E of system at $t_0 = 0$. $(M.E)_{t_0=0} = (K.E + P.Eg + P.Ee)_{t=0}$

$= 0 + 0 + \frac{1}{2}Kx_0^2$

$$\boxed{(M.E)_{t_0=0} = \frac{1}{2}Kx_0^2}$$

* Expression of M.E of system at any t: $(M.E)_t = (K.E + P.Eg + P.Ee)_t$

$= \frac{1}{2}mv_t^2 + 0 + \frac{1}{2}Kx_t^2$

$$\boxed{(M.E)_t = \frac{1}{2}mv_{(t)}^2 + \frac{1}{2}Kx_{(t)}^2}$$

* Differential equation that discribes the variation of $x_{(t)}$:

No friction $\Rightarrow$ M.E of system is conserved

$\Rightarrow M.E_{(t)} = \text{constant}$

$\Rightarrow (M.E)'_{(t)} = 0$

$\Rightarrow \frac{dM.E_{(t)}}{dt} = 0$

$\Rightarrow \left(\frac{1}{2}mv_{(t)}^2 + \frac{1}{2}Kx_{(t)}^2\right)' = 0$

$\frac{1}{2}m(2v_{(t)}v'_{(t)}) + \frac{1}{2}K(2x_{(t)}x'_{(t)}) = 0$

$mv_{(t)}v'_{(t)} + Kx_{(t)}x'_{(t)} = 0$

$\begin{cases} v = x' \\ v' = x'' \end{cases}$

$m\cancel{x'_{(t)}}x''_{(t)} + Kx_{(t)}\cancel{x'_{(t)}} = 0$

$mx''_{(t)} + Kx_{(t)} = 0 \Rightarrow \boxed{x'' + \frac{K}{m}x_{(t)} = 0}$

Solution of diff eq

$x'' + \frac{K}{m}x_{(t)} = 0$

$\Rightarrow x'' + \omega_0^2 x_{(t)} = 0$

* $x_{(t)} = A\sin(\omega_0 t + \phi)$

$\Rightarrow \begin{cases} A = X_m \\ \omega_0^2 = \frac{K}{m} \end{cases} \Rightarrow \boxed{\omega_0 = \sqrt{\frac{K}{m}}}$

* $\vec{T} = -m\vec{g} \Rightarrow T = mg$

$\Rightarrow mg = Kx_0$

* ω_0 : proper pulsation or proper angular frequency
* Proper Frequency : $\boxed{F_0 = \frac{\omega_0}{2\pi}} = \frac{\sqrt{\frac{K}{m}}}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{K}{m}} = F_0}$
* Proper period: $\boxed{T_0 = \frac{2\pi}{\omega_0}} = 2\pi\sqrt{\frac{m}{K}}$

$T_0 = \frac{1}{F_0} = \boxed{2\pi\sqrt{\frac{m}{K}} = T_0}$

* **Determination of ϕ and X_m:**

if $x_{(t)} = X_m \sin(\omega_0 t + \phi)$

$$\begin{cases} x_{(t)} = X_m \sin(\omega_0 t + \phi) \\ V_{(t)} = X_m \omega_0 \cos(\omega_0 t + \phi) \end{cases} \Rightarrow \begin{cases} x_0 = X_m \sin(\phi) \\ V_0 = X_m \omega_0 \cos(\phi) \end{cases} \Rightarrow \begin{cases} \sin(\phi) = \frac{x_0}{X_m} \\ \cos(\phi) = \frac{V_0}{X_m \omega_0} \end{cases}$$

$\Rightarrow$ then substitute X_m and find ϕ by using $\cos^2\phi + \sin^2\phi = 1$ or $\tan\phi = \frac{\sin\phi}{\cos\phi}$

* **Amplitudes of $a_{(t)}$; $x_{(t)}$ & $V_{(t)}$**

$x_{(t)} : -X_m \leq x_{(t)} \leq X_m \Rightarrow \boxed{\text{amplitude} : X_m}$

$V_{(t)} : -\omega_0 X_m \leq V_{(t)} \leq \omega_0 X_m \Rightarrow \boxed{\text{amplitude} : \omega_0 X_m}$

$a_{(t)} : -\omega_0^2 X_m \leq a_{(t)} \leq \omega_0^2 X_m \Rightarrow \boxed{\text{amplitude} : \omega_0^2 X_m}$

* **Time expressions (equation) of energy:**

1) $K.E_{(t)} = \frac{1}{2} m V_{(t)}^2 = \frac{1}{2} m [X_m \omega_0 \cos(\omega_0 t + \phi)]^2 = \frac{1}{2} m \omega_0^2 X_m^2 \cos^2(\omega_0 t + \phi)$

$\Rightarrow K.E_{(t)} = \frac{1}{2} m \frac{K}{m} X_m^2 \cos^2(\omega_0 t + \phi) \Rightarrow \boxed{K.E_{(t)} = \frac{1}{2} K X_m^2 \cos^2(\omega_0 t + \phi)}$

2) $P.E_e = \frac{1}{2} K x_{(t)}^2 = \frac{1}{2} K [X_m \sin(\omega_0 t + \phi)]^2$

$\Rightarrow \boxed{P.E_e = \frac{1}{2} K X_m^2 \sin^2(\omega_0 t + \phi)}$

3) $M.E = K.E_{(t)} + P.E_{e(t)} = \frac{1}{2} K X_m^2 \cos^2(\omega_0 t + \phi) + \frac{1}{2} K X_m^2 \sin^2(\omega_0 t + \phi)$

$\Rightarrow M.E = \frac{1}{2} K X_m^2 [\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi)] =$

$\Rightarrow \boxed{M.E = \frac{1}{2} K X_m^2}$

* $\boxed{T_0 = 2T}$

* **K.E, P.Ee & M.E as fn of x:**

* M.E is constant : $M.E = \frac{1}{2} K X_m^2$
* $P.E_e = \frac{1}{2} K x^2$ of form $y = ax^2$
  $D_f : [-X_m , X_m]$ (parabola passing by O)
* $K.E = M.E - P.E_e = \frac{1}{2} K X_m^2 - \frac{1}{2} K x^2$
  $= \frac{1}{2} K (X_m^2 - x^2)$
* position where $K.E = P.E_e = \frac{M.E}{2}$

$\frac{1}{2} K x^2 = \frac{1}{4} K X_m^2$

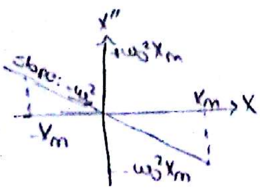
$x^2 = \frac{1}{2} X_m^2 \Rightarrow \boxed{x = \pm \frac{X_m}{\sqrt{2}}}$

* **Graph of x'' as fn of x:**

$x'' + \omega_0^2 x = 0$

$x'' = -\omega_0^2 x$ of form $y = ax$

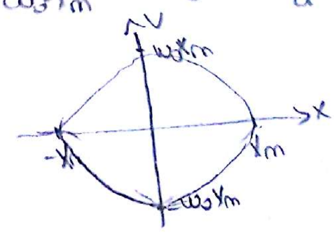
$a = -\omega_0^2$



* **Graph of V as fct of x:**

$\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = 1$

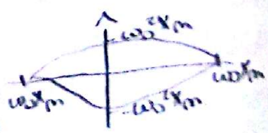
$\frac{x^2}{X_m^2} + \frac{V^2}{\omega_0^2 X_m^2} = 1$ of form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



* **Graph of x'' as fn of V.**

$\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi) = 1$

$\frac{V^2}{X_m^2 \omega_0^2} + \frac{x''^2}{\omega_0^4 X_m^2} = 1$ of form: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



* $E_{\text{supplied by device during } \Delta t} = E_{\text{lost due to function in } \Delta t}$

* $P_{av} = \frac{|\Delta E_m|}{\Delta t} = \frac{|E_{m_2} - E_{m_1}|}{t_2 - t_1} = \frac{\frac{1}{2} K (X_{m_2}^2 - X_{m_1}^2)}{T}$

$P_{av\ supply} = P_{lost} = \frac{|\Delta E_m|}{\Delta t}$

Electromagnetic Induction

* Magnetic flux: $\underset{wb}{\phi} = NBS\cos\theta$ (B in T, S in m^2)

* $i = \frac{e}{R_{total}} \Rightarrow e = Ri$

* Lenz's law:

When an em.f is induced due to a variation in the magnetic flux, the direction of the induced current is such that its electromagnetic effects oppose the cause that is producing it.

- Determination of the variation of i_{ind} by variation of ϕ:

case 1: if ϕ varies in closed circuit $\Rightarrow$ $i_{induced}$ is created
$\Rightarrow B_{induced}$ is created by $i_{induced}$ whose direction is determined by R.H.R

case 2: if $\phi \uparrow \Rightarrow \vec{B}_{ind}$ has opposite direction to $\vec{B}_{external}$
$\Rightarrow$ The direction of i_{ind} is determined by applying R.H.R on $\vec{B}_{ind}$

case 3: if $\phi \downarrow \Rightarrow \vec{B}_{ind}$ has same direction to $\vec{B}_{ext}$
$\Rightarrow$ The direction of i_{ind} is determined by R.H.R on $\vec{B}_{ind}$

- Determination of direction of i_{ind} by using sign of "e":

$e = Ri$ (e & i have same direction)

if $e > 0 \Rightarrow i_{ind} > 0 \Rightarrow i_{ind}$ circulates in positive chosen direction

if $e < 0 \Rightarrow i_{ind} < 0 \Rightarrow i_{ind}$ circulates in opposite direction to chosen positive direction.

* Faraday's law:

$$e = -\frac{d\phi}{dt}$$

* $U_{AB} = ri - e$ (voltage across loop) $U_{AB} = -Ri$ (across resistor)

* Power distribution of (magnet-coil) system:

$P_{electric} = e \times i$

$\left.\begin{array}{l} P_{useful} = U_{AB}\, i \\ + \\ P_{lost} = ri^2 \end{array}\right\} P_{electric}$

$$\boxed{P_{total} = P_{lost} + P_{useful}}$$

Zainab Damerji

Self Induction

Self-induction: is the induction of an electromotive force in a circuit when the current in that circuit is varied.

* Inductance of a coil L:

$$\phi = Li$$

ϕ "Wb", L "H" (Henry), i "A"

i and ϕ always have the same sign $\Rightarrow L > 0$

Expression of the inductance L:

$$\vec{B} = \frac{4\pi \times 10^{-7} N i}{\ell} \vec{n}$$

$$\phi = NSB.\vec{n} \Rightarrow \phi = NS\left[\frac{4\pi \times 10^{-7} N i}{\ell}\vec{n}\right].\vec{n} = \frac{4\pi \times 10^{-7} N^2 S i}{\ell}$$

But $L = \frac{\phi}{i} = \frac{4\pi \times 10^{-7} N^2 S i}{\ell i} \Rightarrow \boxed{L = \frac{4\pi \times 10^{-7} N^2 S}{\ell}}$

* Expression of the self-Induced Electromotive Force:

$$e = -\frac{d\phi}{dt} = -\frac{d(Li)}{dt} \quad (L \text{ is constant})$$

$$\Rightarrow \boxed{e = -L\frac{di}{dt}}$$

But $L > 0 \Rightarrow e$ and $\frac{di}{dt}$ have opposite signs.

* Ohm's law:

$U_{\text{across the coil}} = ri - e$ but $e = -L\frac{di}{dt}$

$$\Rightarrow U = ri - \left(-L\frac{di}{dt}\right) \Rightarrow \boxed{U = ri + L\frac{di}{dt}}$$

Steady state: In the steady state, the current is constant then $\frac{di}{dt} = 0$ the
$\Rightarrow U = ri \Rightarrow$ The coil acts as a pure resistance r since the inductance of the coil has no effect.

Purely inductive coil: If the internal resistance of the coil can be neglected, the coil is said to be purely inductive.
The purely inductive coil acts as a connecting wire in the steady state.

*Role of the coil:

- If $i \uparrow \Rightarrow \frac{di}{dt} > 0 \Rightarrow e < 0$, but i is positive $\Rightarrow e.i < 0$

$\Rightarrow$ The coil acts as a receiver.

- If $i \downarrow \Rightarrow \frac{di}{dt} < 0 \Rightarrow e > 0$, but i is positive $\Rightarrow e.i > 0$

$\Rightarrow$ The coil acts as a generator.

- If i is constant $\Rightarrow \frac{di}{dt} = 0 \Rightarrow e = 0$

$\Rightarrow$ The coil acts as a pure resistor.

*Magnetic Energy stored in a coil:

$U_{BA} = ri - e = ri + L\frac{di}{dt}$ (Multiply by i both sides)

$\Rightarrow iU_{BA} = ri^2 + Li\frac{di}{dt}$

$\boxed{P_{total} = P_{lost} + P_{magnetic}}$

$P_{total} = iU_{BA}$

$P_{lost} = ri^2$

$P_{magnetic} = Li\frac{di}{dt} = -e.i$

*Magnetic energy stored in a coil:

$P = \frac{dW}{dt} \Rightarrow dW = P.dt$

$\Rightarrow W = \int Li\frac{di}{dt}dt = \int Li\,di = \frac{1}{2}Li^2 + C$, $i = 0$ & $W = 0$ $\Rightarrow$ constant $= 0$

$\Rightarrow \boxed{W = \frac{1}{2}Li^2}$

- During the growth of the current, $(W) \uparrow$ and then the coil is storing magnetic energy, so it acts as a reciever.

- During the decay of the current, $(W) \downarrow$ and then the coil is restoring (supplying) its energy to the circuit, so it acts as a generator.

- At the end of the growth of the current, the steady state is attained:

$i = I_{max} \Rightarrow W = \frac{1}{2}Li^2_{max}$

- At the end of the decay of the current, the steady state is attained

$i = 0 \Rightarrow W = 0$

Capacitor

* $C = \frac{q}{U} \Rightarrow q_{(c)} = C_{(F)} \times U_{(V)}$

* $i = \frac{E - U_C}{R}$

* $I_{max} = \frac{E}{R} = I_0$ (at $t_0 = 0$, $U_C = 0$)

* $Q_{max} = C \times U_{C(max)} = CE$

* At end of complete charging : $U_C = U_{AB} = E \Rightarrow i = 0$

} charging of a capacitor

* $i = \frac{-U_C}{R}$

* at $t_0 = 0$: $U_C = E \Rightarrow i = -\frac{E}{R}$

* ent of complete discharging : $U_C = 0 \Rightarrow i = 0$

} discharging of a capacitor

* Energy stored in a capacitor:

$W = \frac{1}{2} C V^2$ $\quad W = \frac{1}{2}\frac{q^2}{C}$ $\quad W = \frac{1}{2} q U$

* τ is time:

$\tau = RC$
$= [\Omega][C]$
$= \frac{[V]}{[A]}\frac{[C]}{[V]}$
$= \frac{[V]}{[A]}\frac{[A][S]}{[V]}$
$= [S] \Rightarrow \tau$ is a time

* Charging of a capacitor:

$U_C + Ri = E$

① diff eq that discribes the variation of U_C:

$\frac{dU_C}{dt} + \frac{U_C}{RC} = \frac{E}{RC}$

Solution: $U_C = A + Be^{-\frac{t}{\tau}}$ $\Rightarrow \boxed{U_C = E(1 - e^{-\frac{t}{\tau}})}$

② Expressions of $i(t)$ & $U_{R(t)}$ from expression U_C

i) Expression of $i(t)$:

$i = C\frac{dU_C}{dt} \Rightarrow i = \frac{E}{R} e^{-\frac{t}{\tau}} \Rightarrow \boxed{i = I_{max} e^{-\frac{t}{\tau}}}$

ii) Expression of U_R:

$U = Ri \Rightarrow U_R = E e^{-\frac{t}{\tau}}$

③ Equation of tangent to U_C at $t_0 = 0$

$U_C = at$: $a = \frac{E e^{-0}}{\tau}$

eq of tangent : $U_C = \frac{E e^{-0}}{\tau} = \frac{E}{\tau}$

* to prove (tangent of U_C at $t=0$) intersect the asymptote:

eq of tangent : $U_C = \frac{E}{\tau}$

eq of asymptote: $U = E$

$\Rightarrow U_C = U_C \Rightarrow t = \tau$

④ Diff eq of q, i & U_R:

* diff eq of q: $\frac{dq}{dt} + \frac{q}{RC} = \frac{E}{R}$ — solution: $q = Q_{max}(1 - e^{-\frac{t}{\tau}})$
* diff eq of i: $\frac{di}{dt} + \frac{i}{RC} = 0$ — solution: $i = I e^{-\frac{t}{\tau}}$
* diff eq of U_R: $\frac{dU_R}{dt} + \frac{U_R}{RC} = 0$ — solution: $U_R = (U_R)_{max}\, e^{-\frac{t}{\tau}}$, $(U_R)_{max} = R I_{max}$

* Discharging of a Capacitor:

$U_C - Ri = 0$

① diff eq of U_C:

$\frac{dU_C}{dt} + \frac{U_C}{RC} = 0$ — Solution: $U_C = A e^{-\frac{t}{\tau}} \Rightarrow U_C = E e^{-\frac{t}{\tau}}$

② Expressions of i, q & U_R using expression of U_C:

i) expression of q:

$q = C \times U_C = C \times E e^{-\frac{t}{\tau}} \Rightarrow \boxed{q = Q_{max}\, e^{-\frac{t}{\tau}}}$

ii) expression of i:

$i = -\frac{dq}{dt} = -\frac{C\,dU_C}{dt} \Rightarrow \boxed{i = \frac{E}{R} e^{-\frac{t}{\tau}}}$

iii) expression of U_R:

$U_R = Ri \Rightarrow \boxed{U_R = E e^{-\frac{t}{\tau}}}$

③ Equation of tangent to U_C at $t_0 = 0$

$U_C = at + b$

$a = \text{slope} = \left(\frac{dU_C}{dt}\right)_{t=0} \Rightarrow a = -\frac{E}{\tau}$

Use pt $(0, E) \Rightarrow E = b$

$\Rightarrow$ equation of tangent: $\boxed{U_C = -\frac{E}{\tau}t + E}$

* to prove ∩ between tangent to U_C & any state $U_C = 0$

eq of tangent: $U_C = -\frac{E}{\tau}t + E$

eq of any state: $U_C = 0$

$U_C = U_C$

$\Rightarrow \boxed{t = \tau}$

④ Diff eq of q, i & U_R:

* diff eq of q: $\frac{dq}{dt} + \frac{q}{RC} = 0$ — solution: $q = Q_{max}\, e^{-\frac{t}{\tau}}$
* diff eq of i: $\frac{di}{dt} + \frac{i}{RC} = 0$ — solution: $i = I_{max}\, e^{-\frac{t}{\tau}}$
* diff eq of U_R: $\frac{dU_R}{dt} + \frac{U_R}{RC} = 0$ — solution: $U_R = U_{Rmax}\, e^{-\frac{t}{\tau}}$

Alternating Sinsoidal Current

- The oscilliscope always measure U_{phase_ground}

* $\boxed{U(t) = U_m \sin(\omega t + \varnothing)}$

* Characteristics of alternating sinsoidal voltage:
  - Period T
  - Frequency F
  - amplitude U_m

* $\boxed{T = S_h \times x}$ (S_h may be donated by V_b or D_b) } Period
  "s" s/div

* Frequency : $\boxed{F = \frac{1}{T} \text{ (Hz)}}$

* Angular frequency : $\boxed{\omega = 2\pi F}$ $\boxed{F = \frac{\omega_0}{2\pi}}$ $\boxed{\omega = \frac{2\pi}{T}}$

* Amplitude (maximum voltage) : $\boxed{U_m = S_v \times y}$

* alternating sinsoidal voltage gives birth to an alternating sinsoidal current:
  $U = U_m \sin(\omega t + \varnothing)$ $\longrightarrow$ $\boxed{i = I_m \sin(\omega t + \varnothing')}$
  (u & i have same form & frequency).

* Calculation of Im:
  $U_R = Ri \Rightarrow (U_R)_{max} = R I_{max} \Rightarrow \boxed{I_{max} = \frac{(U_R)_{max}}{R}}$ $((U_R)_{max} = S_v \times y)$

* Phase difference between U_g & i:
  $\boxed{\Delta\phi = \varnothing_2 - \varnothing_1}$

- if $\varnothing_1 = \varnothing_2 \Rightarrow \Delta\varnothing(U_g/i) = 0$ (U_g & i are inphase)
  $\Delta\varnothing = 0$
  (U_g & i) attain max, min & vanishes at same time

- if $\varnothing_i > \varnothing_{U_g} \Rightarrow \Delta\varnothing(i/U_g) > 0$
  $\Rightarrow$ i leads U_g (U_g lags behind i)
  (max or min of i appears before U_g)
  $\boxed{|\Delta\varnothing| = 2\pi \frac{d}{D}}$ $\longleftarrow$ if + (i leads U_g) (i/U_g)
  if − (U_g lags behind i) (U_g/i)

- if $\varnothing_i < \varnothing_{U_g} \Rightarrow \Delta\varnothing(i/U_g) < 0 \Rightarrow$ i lags behind U_g (U_g leads i)
  $|\Delta\varnothing| \longleftarrow$ if − (i/U_g) [i lags behind U_g]
  if + (U_g/i) [U_g leads i]

  if $U_g = U_m \sin(\omega t + \varnothing_1)$
  $\Rightarrow \boxed{i = I_m \sin(\omega t + \varnothing_1 - 2\pi \frac{d}{D})}$

* Effective voltage: $\boxed{U_{eff} = \frac{U_m}{\sqrt{2}}}$ (measured by voltmeter)
* Effective current: $\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$ (measured by ammeter)
* $R = \frac{U_{R\,eff}}{I_{eff}}$
* Ohm's law:

① Case of Resistor: In an alternating sinsoidal circuit, i & U_R are always inphase
$\Delta\phi(U_R/i) = \Delta\phi(i/U_R) = 0$

② case of coil: In an A.C, U_{coil} leads i $\Rightarrow \Delta\phi(u_{coil}/i) > 0 = \frac{2\pi d}{D}$
for a purely inductive coil: U_{coil} leads i by $\frac{\pi}{2} \Rightarrow \Delta\phi(u_{coil}/i) = +\frac{\pi}{2}$

③ Case of capacitor: U_c lags behind i by $\Delta\phi = \frac{2\pi d}{D} = \frac{\pi}{2}$
$\Delta\phi(u_c/i) = -\frac{\pi}{2}$ $\Delta\phi(i/u_c) = \frac{\pi}{2}$

* Remark: $i = \frac{dq}{dt} = C\frac{du}{dt} \Rightarrow u_C = \frac{1}{C}\int i\,dt \Rightarrow u_{coil} = L\frac{di}{dt} + ri \Rightarrow u_R = Ri$

* (R_C) circuit: $U_g = U_R + U_C$ (U_g lags behind i)
* (R_L) circuit: $U_g = U_R + U_L \Rightarrow U_g$ leads i
* Case of RLC series circuit:

$F_0 = \frac{1}{2\pi\sqrt{LC}}$

- For values $F_{generator} < F_0$; i leads U_g
  $\Delta\phi_{i/U_g} > 0 \Rightarrow$ circuit is capacitor
- $F_{generator} = F_0$ (i & U_g are inphase)
  $\Rightarrow$ circuit is resistive.
- $F_{generator} > F_0$: i lags behind U_g (U_g leads i)
  $\Rightarrow$ circuit is inductive

* The current i varies with the the frequency "f" of L.F.G.
* When $f = f_0$; the circuit undergoes a physical phenomenon called circuit_Resonance
* At current Resonance:

- $F_{generator} = F_{0(R\_L\_C)} = \frac{1}{2\pi\sqrt{LC}}$
- U_g & i are inphase. $\Delta\phi = 0$
- The circuit is purely resistive
  $(U_g) = U_{(R+r)} \Rightarrow U_g = (R_{total})\, i$
  $(U_m)_g = R_{total}\, I_{max} \Rightarrow (U_{eff})_g = R_{total}\, I_{eff}$
  $\Rightarrow \boxed{I_{eff} = \frac{U_{eff}}{R_{total}}}$
- $LC\omega^2 = 1$
- I_m of I_{eff} attains their maximum values

* Average Power and Power factor:

- $P_{average} = (U_{eff})_X \times (I_{eff})_X \times \cos(\phi_{i/U_X})$
  (delievered or consumed by X)
- $P_{app} = U_{eff} \times I_{eff}$
- $\cos\phi = \frac{P_{av}}{P_{app}}$ $P_{av} = P_{app} \cdot \cos\phi$
- if X is a resistor : $\phi_{i/U_R} = 0$
  $\Rightarrow \cos\phi_{i/U_R} = 1$
  $\Rightarrow (P_{av})_R = (U_{eff}) \times I_{eff} \times 1$
  $\Rightarrow (P_{av})_R = U_{eff} \cdot I_{eff} \begin{cases} RI^2_{eff} \\ \frac{U^2_{eff}}{R} \end{cases}$
- If X is a capacitor :
  $\phi_{i/U_C} = +\frac{\pi}{2} \Rightarrow \cos\phi = 0$
  $\Rightarrow (P_{av})_{capacitor} = 0$
- If X is purely inductive coil
  $\phi_{i/U_C} = -\frac{\pi}{2} \Rightarrow \cos\phi = 0$
  $\Rightarrow P_{av} = 0$

* The loss in energy produces only in the case of resistor.

Diffraction

* Characteristics of the fringes:
- Alternate Bright and Dark.
- Perpendicular to the direction of the slit.
- The width of the central bright fringe is double (twice) the width of the lateral bright fringe.
- The intensity of the bright fringe decrease as we go away from the central bright fringe.

* Diffraction of light: duration of light path without being reflected or ~~reflected~~ refracted.

* $\underset{(\text{in medium})}{V} = \dfrac{c}{n_{medium}}$ (speed of light)

$\upsilon_{light} = \dfrac{c}{\lambda}$ (in vaccum)

$\upsilon = \dfrac{V}{\lambda}$ (in medium)

υ_{light} doesn't change with medium

$\Rightarrow \boxed{\lambda_{medium} = \dfrac{\lambda_{vaccum}}{n}}$

* In vaccum

ultra violet $\rightarrow$ $0.4\,\mu m \leq \lambda \leq 0.8\,\mu m$ $\rightarrow$ Infra red

In medium

$$\frac{0.4\mu m}{n} \leq \lambda_{medium} \leq \frac{0.8\,\mu m}{n}$$

* Types of diffraction:
  1_ Fraunhofer's diffraction (screen is far from slit)
  2_ Fresen l's diffraction (screen is close from slit).

* Angular position of a dark fringe of order K.

$\sin \theta_K = K\dfrac{\lambda}{a}$

but since θ is very small

$\Rightarrow \boxed{\theta_{K(rd)} = \dfrac{K\lambda}{a}}$

* Linear position of the center dark fringe:

$O_{dK} = Y_K$

$Y_K = \frac{K\lambda D}{a}$

$Y_K = \theta_{K(d)} \cdot D$

* angular width of C.B.F "α"

$\alpha_{(d)} = \frac{2\lambda}{a}$

angular width

α → proportional to λ
→ inversely proportional to a.

* Linear width of C.B.F "L"

$\boxed{L = \frac{2\lambda D}{a}}$

$\boxed{L = D\alpha_{(d)}}$

L → proportional to λ & D
→ inversely proportional to a.

* Width of a B.F:

$\ell_{B.F} = X_{K+1} - X_K$

$= (K+1)\frac{\lambda D}{a} - \frac{K\lambda D}{a}$

$= \frac{\lambda D}{a} = \frac{L}{2} = \frac{\ell_{C.F}}{2}$

* For a circular hole:

$\alpha_{C.F} = 2\theta_1 = 2(1.22\frac{\lambda}{a})$

$L_{C.F} = 2(1.22\frac{\lambda D}{a})$

* Huygen's principle: Each point in the slit acts as a secondary source of wavlets and the diffraction pattern formed on the screen is due to the superposition of these secondary ~~sources~~ waves.

*

*

Interference of Light

* Path Difference of M: $\delta_{(M)} = S_1M - S_2M$

if $\delta_{(M)} = S_1M - S_2M = K\lambda$
$\Rightarrow M \in B.F$ of order K (inphase)

if $\delta_{(M)} = S_1M - S_2M = \frac{(2K+1)\lambda}{2} = \frac{odd\,\lambda}{2}$
$\Rightarrow M \in D.F$ of order K (out of phase)

nature of M

* Definition of Interference: It is the superposition of 2 or more waves having same frequency in the same region, even though, their amplitude and phase are different.

* optical path: $(AB) = nAB$ (in air or vaccum $n=1$)

* Description of interference figure in yong's experiment:
The interference figure that is observed on screen (E) is formed of fringes that are:
- Rectilinear
- parallel to the slits
- Equidistant
- Alternating bright and dark

* Cause of interference:
The interference is due to the superposition of luminous waves issued from S and diffracted by slits S_1 & S_2.

* Reason behind the formation of interference fringes:
- The bright fringes are due to the superposition of waves inphase.
- The dark fringes are due to the superposition of waves out of phase.

* Conditions of interference:
To obtain a system interference (stable interference), the sources S_1 & S_2 must be:
- Synchronized (emitts waves of same frequency)
- Coherent (having constant phase difference $\Rightarrow$ originating from same main source).

* The optical path difference at point M is:

$$\delta_{(M)} = (SS_2M) - (SS_1M)$$
$$= \cancel{(SS_2)} + (S_2M) - \cancel{(SS_1)} - (S_1M)$$
$$= (S_2M) - (S_1M)$$
$$= n(S_2M) - n(S_1M)$$
$$= n(S_2M - S_1M)$$

in air or vaccum: $n=1$
$\Rightarrow \delta_{(M)} = S_2M - S_1M$
$= d_2 - d_1$

Scanned by CamScanner

* Geometrical determination of the optical path difference:

$\delta(M) = S_2M - S_1M \approx S_2H \Rightarrow \delta(M) = S_2H$

The position of M is given by its abscissa:

$x_M = \overline{OM}$

In Δ S_1S_2H : $\sin\theta = \frac{S_2H}{S_1S_2} = \frac{\delta(M)}{a}$

In Δ O_1OM : $\tan\theta = \frac{OM}{OO_1} = \frac{x(M)}{D}$

$\theta \lll \Rightarrow \sin\theta = \tan\theta$

$\frac{\delta(M)}{a} = \frac{x(M)}{D} \Rightarrow \boxed{\delta(M) = \frac{x(M) \cdot a}{D}}$

* Position of center of B.F:

if M is the center of B.F of order K

$\Rightarrow \delta(M) = K\lambda$

$\frac{x(M) \cdot a}{D} = K\lambda \Rightarrow \boxed{x(M) = \frac{K\lambda D}{a}}$

* Position of center of D.F:

if M is the center of D.F of order K

$\delta(M) = \frac{(2K+1)\lambda}{2}$

$\frac{x(M) \cdot a}{D} = \frac{(2K+1)\lambda}{2} \Rightarrow \boxed{x(M) = \frac{(2K+1)\lambda D}{2a}}$

* Position of the central B.F:

If M is the center of central B.F

$\delta(M) = 0$

$\frac{x(M) \cdot a}{D} = 0 \Rightarrow x(M) = 0$

$\Rightarrow$ M confoundes with O.

* Interfringe: is the distance separating the centers of two successive fringes of same nature.

* Expression of i. $i = (x_{K+1} - x_K)$ O.F or B.F

$\boxed{i = \frac{\lambda D}{a}}$

$\boxed{\lambda_{medium} = \frac{\lambda_{air}}{n}}$

when $S_2M > S_1M \Rightarrow$ M moves up

$S_2M < S_1M \Rightarrow$ M moves down

Photoelectric Effect

*Photoelectric effect is the emission phenomenon of extracting an electron from a metal and this metal is illuminated with a convenient light.

*Planck's postulate: The exchange of energy between matter and radiation is quantized.

* characteristics of a photon:

- Has zero mass
- Has no charge
- moves with speed with light
- carries well determined energy

* $E = h\nu = h\frac{c}{\lambda}$

$\lambda_0 = \frac{c}{\nu_0}$ $\quad\quad$ $\nu_{radiation} \geq \nu_0 \iff$ photoelectric effect takes place

$\lambda \leq \lambda_0$

*Work function: It is the minimum energy necessary to extract an electron from a metal or the energy binding the electron to the metal.

$W_0 = h\nu_0 = h\frac{c}{\lambda_0}$ $\quad\quad$ $E_{photon} \geq W_0$

* Interaction: photon - Metal:

- $E_{photon} < W_0 \Rightarrow$ photon is not absorbed
  $\Rightarrow$ No electrons is ejected $\Rightarrow$ No photoelectric effect

- $E_{photon} = W_0 \Rightarrow$ photon is absorbed, the electron is liberated from the metal but without speed, it remains on the surface metal.

- $E_{photon} > W_0 \Rightarrow$ photon is absorbed
  In this case part of E_{photon} serves to liberate the electron from the metal, and the remaining part is carried as a maximum kinetic energy of the ejected electron.

* Einstein Relation:

$E = W_0 + K.E$

$h\nu = h\nu_0 + \frac{1}{2} m_e V_{max}^2$

* $P_r = \frac{N.\ E_{photon}}{1sec}$

* $P_{beam} = \frac{E_{beam}}{\Delta t} = \frac{Nh\nu}{\Delta t}$

* $E_{total} = Nh\nu = Nh\frac{c}{\lambda}$

* Effective photons: The photons that are capable to extract an electron from metal and so producing electric emission.

* $\eta_{quantum} = \frac{N_{eff}}{N_{total}} = \frac{N_{extracted\ e^-}}{N_{total}}$

* $n = \frac{P_{eff}}{P_{incident}}$

* $I = \frac{|\Delta q|}{\Delta t} = \frac{N_{e^-} \cdot e}{\Delta t}$

$N_{e^-s} = N_{eff\ photons}$

* $K.E_{max} = h.\upsilon - W_0$

* $E_{incident} = W_0 + (K.E)_{max\ extract\ e^-}$

* $K.E_{max} = E_{incident\ photon} - W_0$

* $K.E_{max} = \frac{hc}{\lambda} - W_0$

* $\upsilon_0 = \frac{W_0}{h}$

* The threshold frequency υ_0 of a metal is the minimum frequency necessary to extract an electron from a metal (produce photoelectric effect).

* if $P_{beam} \uparrow$; $N_{incident\ photons} \uparrow \Longleftrightarrow$ no of extracted $e^- \uparrow \Longleftrightarrow I \uparrow$

* Intensity of beam $\uparrow \Longleftrightarrow E_{photon} \uparrow \Longleftrightarrow K.E_{extracted\ e^-} \uparrow$

* $P_{rec} = \frac{E_{recieved}}{t}$

* $P = nE$

* $r = \frac{N_{eff}}{N_{rec}}$

* $N_{rec} = \frac{P_r \times t}{E_{photon}}$

* $\lambda_0 = \frac{hc}{W_0}$

* The threshold wavelength λ_0 of a metal is the maximum wavelength that can extract an electron from a metal (produce photoelectric effect)

The atom

- $E_{photon} = E_i - E_f = \frac{hc}{\lambda} = h\upsilon$
- Balmer series: $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ $n \geqslant 3$

- H_α: corresponds to transition from $n=3$ to $n=2$
  H_β: corresponds to transition from $n=4$ to $n=2$
  H_γ: corresponds to transition from $n=5$ to $n=2$
  H_δ: corresponds to transition from $n=6$ to $n=2$

* $\frac{1}{\lambda} = \frac{1}{hc}(E_i - E_f)$

* For Balmer series: $E_n = \frac{-Rhc}{n^2} = \frac{-13.6}{n^2}$

* Spectral series of hydrogen atom:

Layman series: from $n \geqslant 2$ to $n=1$ $\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$

Balmer series: from $n \geqslant 3$ to $n=2$ $\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

Paschen series: from $n \geqslant 4$ to $n=3$ $\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$

Brackett series: from $n \geqslant 5$ to $n=4$ $\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$

Pfund series: from $n \geqslant 6$ to $n=5$ $\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$

* $E_{ionized} = E_\infty - E_i$

* Excitation of an external photon:

$\underset{extracted}{(K.E)} = E_{photon} - E_{ionized}$

- if $E_i + E_{photon} = E_j$
  where E_j is a higher level
  $\Rightarrow$ atom will jump upward from E_i to E_j (photon is absorbed)

- if $E_i + E_{photon} = 0 = E_\infty$
  $\Rightarrow$ photon is absorbed
  $\Rightarrow$ atom is excited to E_∞ $(K.E = 0)$
  $\Rightarrow$ atom is ionized

- if $E_i + E_{photon} = a < 0$
  In this case photon is not absorbed and atom remains at level E_i

- if $E_i + E_{photon} = a > 0$
  $\Rightarrow$ photon is absorbed
  part of E_{photon} is needed to excite atom to E_∞ and the remaining part is carried by an extracted electron as K.E to get out the atom.

* Excitation by the collision of an external e^-:

- if $(K.E)_{e^-} + E_i = E_j$
  where E_j is one of the energy levels
  ⇒ atom is excited to energy level E_j and the electrons loss totally its K.E.

- if $(K.E)_{e^-} + E_i = $ value < 0
  with $E_i < E < E_j$
  ⇒ atom is excited to E_j and the electrons continues with the remaining part of its K.E

- if $(K.E_{e^-}) + E_i < E_{i+1}$
  where E_{i+1} is the energy level just after E_i
  ⇒ K.E is not enough to excite atom to E_{i+1}
  ⇒ electrons continues with its K.E and the atom remains at E_i

* Remark: upward transition ⇒ excitation ⇒ absorption spectrum
  downward transition ⇒ de-excitation ⇒ emission spectrum.

* missing line: due to the absorption by its corresponding photon by atoms of gaz.

* Remark: The missing spectral line in absorption spectrum are those obtained in the emission spectrum for the same gaz atoms.

* Ritz formula:

$$E_{(4;1)} = E_{(4;3)} + E_{(3;2)} + E_{(2;1)}$$

$$h\nu_{(4;1)} = h\nu_{(4,3)} + h\nu_{(3;2)} + h\nu_{(2;1)}$$

$$\nu_{(4;1)} = \nu_{(4;3)} + \nu_{(3,2)} + \nu_{(2;1)}$$

$$\frac{1}{\lambda_{(4;1)}} = \frac{1}{\lambda_{(4,3)}} + \frac{1}{\lambda_{(3,2)}} + \frac{1}{\lambda_{(2;1)}}$$

Atomic Nucleus

* Total Number of nucleus = number of protons + number of neutrons

$$A = Z + N$$

↑ mass no

* Nucleide : $^{A}_{Z}X$

* Chemical elements: set of atoms having same atomic number Z.

* Isotops: having same atomic number Z but of different mass number A.

Note: Yale 3nda 23la mass no, hie l btkoun more abundant.

* The one has greatest abundance is dominant.

* $V_{nucleus} = \frac{4}{3}\pi r^3$ (volume of whole sphere)

or = volume of all nucleons $= A \times \frac{4}{3}\pi r_0^3$

* Radius of nucleide r: $r = r_0 A^{\frac{1}{3}}$

r is proportional to $A^{\frac{1}{3}}$

as $A\uparrow$, $r\uparrow$

* $1\,fm = 10^{-15}\,m$

* $r_{atom} \ggg r_{nucleus}$ => there is a big empty space (enormous) in atom occupied by electron cloud.

* $m_{nucleus} = A \times m_{\text{average mass of one nucleus}}$

* $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (A^{\frac{1}{3}} r_0)^3$ => $V_{nucleus} = \frac{4}{3}\pi r_0 A^{\frac{1}{3}} = \frac{4}{3}\pi r_0^3 A$
$= \frac{4}{3}\pi r^3$

* $\rho_{nucleus} = \frac{m_{nucleus}}{V_{nucleus}} = \frac{A \times m_{av}}{\frac{4}{3}\pi r_0^3 A} = \frac{m_{av}}{\frac{4}{3}\pi r_0^3}$

* $1\,MeV = 1.602 \times 10^{-13}\,J$

* $m\,X_{\text{nucleus collected}} > m_{\text{nucleus separated}}$

$m\,X_{collected} > m_{protons} + m_{neutrons}$

$Z.m_{pr} + N.m_{neutrons}$

$\Rightarrow m_X < Z.m_p + (A-Z)\,m_{neutrons}$

* Mass defect of Nucleus:

$$\Delta m = [Z.m_{proton} + (A-Z)m_{neutron}] - m_X$$

* Binding energy of neucleus: E_B

$$E_B = \text{mass defect} \times c^2$$
$$= \Delta m c^2$$
$$= [Z.m_p + (A-Z)m_{neutrons} - m_X] \times c^2$$

* Binding energy per neucleon:

$$E_b = \frac{E_B}{A} = \frac{[Z.m_p + (A-Z)m_{neutron} - m_X]c^2}{A}$$

* Stability of Nucleus:

The most stable nuclei are those of larger binding energy per nucleon E_b

* Interactions inside Neucleus:

- gravitational forces $\vec{F_g}$ (between Neucleons)
  These forces are attractive
- Electric forces $\vec{Fe}$ (between protons)
  ⇒ Repulsive force (p^+'s has same charge)
- Nucleon forces $\vec{F_n}$ between two nucleons (attractive)
  (independent of nature of nucleus)

$$\|\vec{F_n}\| \ggg \|\vec{Fe}\| \ggg \|\vec{F_g}\|$$

* The Binding energy of a nucleus is the minimum energy necessary to Break down the nucleus into its individual nucleons.

Radioactivity

* B

- Radioactivity: it is a spontaneous tranformation of a non-stable nucleus $^{A}_{Z}X$ into a stable nucleus $^{A'}_{Z'}Y$ by emitting a particle and liberating an electron.

$$^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{A''}_{Z''}\text{particle} + \text{energy}$$

parent nucleus — daughter nucleus

* Law of conservations:

a) Law of conservation of Mass number:

$$(\Sigma A)_{before} = (\Sigma A)_{after}$$

b) Law of conservation of charge number:

$$(\Sigma Z)_{before} = (\Sigma Z)_{after}$$

c) Law of conservation of total energy:

$$\Sigma(mc^2 + K.E)_{before} = \Sigma(mc^2 + K.E)_{after} + E_{radiation}$$

* Emission of γ radiation

- properties of γ radiation:
  - has zero mass
  - has no charge
  - has speed of light $c = 3\times10^8$
  - $E_\gamma = h\nu = \frac{hc}{\lambda}$
  - ionized and very penetrating

* - When $^{A'}_{Z'}Y$ is created in an excited state. It de-excites directly or progressively to the ground state by emitting a photon (γ).

- Equation of de-excitation:

$$Y^*_{(excited\ state)} \longrightarrow Y_{(ground\ state)} + \gamma$$

- emission of γ is due to mass defect between Y^* & Y

$$E_\gamma = \Delta mc^2 = [m_{Y^*} - m_Y]c^2$$

* Disintegration α (Emission):

- Symbol: ^4_2H : Helium Nucleus-4
- properties:
  - It is ionizing particle
  - It has a very small speed compared to that of light
  - It has weak penetrating power
  - Very dangerous due to it's ionization effect.
  - It is positively charged particle.

- Production: The α emission is produced by the heavy radioactive (non-stable) nuclei
- Effect: α provokes the ionization of the matter it meets.
- Nuclear reaction: $^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{4}_{2}He$
- Mass defect of reaction: $\Delta m_{reaction} = m_{before} - m_{after} = m_X - (m_Y + m_\alpha) > 0$

$\Rightarrow$ there is a loss in mass $\Rightarrow$ liberation of energy takes place

- Energy liberates:

$E_{lib\,reaction} = \Delta m \times c^2 = [m_X - (m_Y + m_\alpha)] c^2$

- form of E liberates: $E_{lib} = \Delta m c^2 = K.E_Y + K.E_\alpha - K.E_X + E_\gamma$

In general: $(K.E)_X \simeq 0$ (at rest)

$(K.E)_Y = 0$ (created at Rest)

$E_{lib} = K.E_\alpha + E_\gamma$

if no γ emission $= E_\gamma = 0 \Rightarrow E_{lib} = K.E(\alpha)$

* β^- emission (disintegration):

- symbol: ${}^{0}_{-1}e$: electron created inside the nucleus.
- Properties of β^-: - Displaces with a very high speed
  - It is less ionizing than particle α
  - It has a high penetrating power.
  - It is negatively charged.
- Production: the β^- disintegration is possessed by the non-stable (radioactive) nuclei that has excess (rich) of neutrons $(N \geqslant Z)$
- Reaction: ${}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{0}_{-1}e + {}^{0}_{0}\gamma$
- antineutrino: to accompany the emission of β^-

Antineutrino ${}^{0}_{0}\bar{\nu}$: - has no charge
  - has the speed of light in vacuum C
  - It has an enormous penetrating power.
  - It has a very light (weak) mass.
  - It doesn't interact with matter.
- Global reaction of β^- decay: ${}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu} + {}^{0}_{0}\gamma$
- equation of formation of β^- inside nucleus:

${}^{1}_{0}n \longrightarrow {}^{1}_{1}p + {}^{0}_{-1}e + {}^{0}_{0}\bar{\nu}$

- Mass defect of reaction: $\Delta m = m_b - m_a = m_X - (m_Y + m_{{}^{0}_{-1}e}) > 0$

$\Delta m > 0 \Rightarrow$ reaction liberates energy (exoenergetic).

- liberated energy reaction: $E_{lib} = \Delta m c^2 = [m_X - (m_Y + m_{{}^{0}_{-1}e})] c^2$
- Form of E liberated: $E_{lib} = \Delta m c^2 = K.E_Y + K.E_{{}^{0}_{-1}e} + K.E_{{}^{0}_{0}\bar{\nu}} + E_{{}^{0}_{0}\gamma}$

if ${}^{A'}_{Z'}Y$ is created in ground state: $E_\gamma = 0$

" " " " at Rest $\Rightarrow K.E(Y) = 0$

* β^+ decay:

- symbol: ${}^{0}_{+1}e$
- properties: - has very high speed
  - has very high K.E
  - has very large penetrating power
  - It is positively charged.
- production: β^+ disintegration is possessed by the non-stable Nuclei that has excess (rich) of protons $(Z \geqslant N)$

Radioactivity 2:

* β^+ decay:
- Equation: ${}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{0}_{+1}e + {}^{0}_{0}\gamma$
- Neutrino: to accompany the emission of β^+
  ${}^{0}_{0}\nu$: - speed: c
  - very light mass: $m \simeq 0$
  - has no charge
  - It doesn't interact with matter
- Global rxn of β^+ disintegration:
  $${}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + {}^{0}_{+1}e + {}^{0}_{0}\nu + {}^{0}_{0}\gamma$$

- equation of transformation of proton into neutrons inside Nuclei:
  $${}^{1}_{1}p \longrightarrow {}^{1}_{0}n + {}^{0}_{+1}e + {}^{0}_{0}\nu$$

- Form of E liberated:
  $$E_{lib} = K.E_Y + K.E_{{}^{0}_{+1}e} + E_{{}^{0}_{0}\nu} + E_{{}^{0}_{0}\gamma}$$
  if ${}^{A'}_{Z'}Y$ is created at Rest $K.E(Y) = 0$
  " " " " " ground state: $E_\gamma = 0$

* Radioactive Family (series):
  Rxn: ${}^{A}_{Z}X \longrightarrow {}^{A''''}_{Z''''}Y + n\alpha + k\beta^- + m{}^{0}_{0}\bar{\nu} \ldots$

* law of Radioactive decay:
  $$N_{(t)} = N_0 e^{-\lambda t}$$

* expression of their graphs.
  $N = N_0 e^{-\lambda t} \Rightarrow \ln\left(\frac{N}{N_0}\right) = \ln e^{-\lambda t} \Rightarrow \ln N - \ln N_0 = -\lambda t$
  $\Rightarrow \ln N = -\lambda t + \ln N_0$
  $y = at + b \quad$ slope $= -\lambda$

* $N = \frac{m}{M} \times N_A$ (number of nuclei N)
  $$N = \frac{m(Kg)}{A.u(Kg)}$$

* law of radioactive decay of mass:
  $N_0 = \frac{m_0}{M} \times N_A$ (at $t_0 = 0$)
  $m(t) = m_0 e^{-\lambda t} \qquad m = \frac{m_0}{2^n}$

* $T = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$

* $N_{rem} = \frac{N_0}{2^n}$

* $n = \frac{t}{T} \longrightarrow t = nT$

* Activity of Radio element:

- Definition: the activity of a radioactive sample $^{A}_{Z}X$ is the number of disintegration per unit of time.
- unit : Bq ($1Bq = 1 dist/sec$) ($1Ci = 3.7 \times 10^{10} Bq$)
- Formula: $A = -\frac{dN}{dt}$ $A_{av} = -\frac{\Delta N}{\Delta t}$

$\Rightarrow \boxed{A = \lambda N}$

* law of radioactive decay for A:

$A(t) = A_0 e^{-\lambda t}$

* $A = \frac{A_0}{2^n}$

- Remarks:

1) $N_{remained} = N_0 e^{-\lambda t}$ $\Rightarrow N_{disintigrated} = N_0 - N_{rem}$
$= N_0 - N_0 e^{-\lambda t}$
$= N_0 (1 - e^{-\lambda t}) = N_{formed\ for\ Y}$

2) $N_{dist} = A \times t = \lambda N_0 t$

3) $A = -\frac{dN}{dt}$

$(A)_{tp} = ??$ $(A)_{tp} = \left(-\frac{dN}{dt}\right)_{tp} = (-\text{slope of tan})_{tp}$

$E'_{lib} = E_{lib} \times N_0$

$m = \frac{E'_{lib}}{E_{lib}}$

$E = P \times \Delta t$

$E = N \times E_{lib}$ av of one nucleus

* Role of a moderator: will help in reducing their speed so as to provoke more much reactions.

Nuclear Reactions

*Definition: A nuclear reaction said to be provoked (stimulated) if it take place under the intervention of an external agent (factor).

1) General transmutation:

Transformation of a nucleus X into another nucleus Y after bombarding by projectile nucleus.

$$\text{Projectile} + \underset{\text{target}}{X} \rightarrow Y + \text{particle}$$

2) Nuclear fission:

- Definition: Fission is a provoked nuclear reaction during which a heavy nucleus $^{A}_{Z}X$ is divided into 2 daughter nuclei $^{A_1}_{Z_1}Y$ & $^{A_2}_{Z_2}Y$ under the impact of a neutron $^{1}_{0}n$

- Nuclear reaction: $^{1}_{0}n + ^{A}_{Z}X \rightarrow ^{A_1}_{Z_1}Y + ^{A_2}_{Z_2}Y + x\,^{1}_{0}n + \text{radiation}$.

- law of conservation of mass number: $(\Sigma A)_{before} = (\Sigma A)_{after}$

- law of conservation of charge number: $(\Sigma Z)_{before} = (\Sigma Z)_{after}$

- law of conservation of total energy: $\Sigma(mc^2 + K.E)_{before} = \Sigma(mc^2 + K.E)_{after} + E_{rad}$.

- Mass defect of reaction: $\Delta m = m_{before} - m_{after}$

$$= (m_{^1_0n} + m_{^A_ZX}) - (m_{Y_1} + m_{Y_2} + x\,m_{^1_0n}) > 0$$

$\Rightarrow$ reaction is exoergic $\Rightarrow$ reaction liberates energy

- Energy liberated by reaction: $E_{lib} = \Delta m c^2$

$$= [(m_{^1_0n} + m_X) - (m_{Y_1} + m_{Y_2} + x\,m_{^1_0n})]c^2$$

form of liberated energy: $E_{lib} = K.E_{Y_1} + K.E_{Y_2} + xK.E_{(^1_0n)} + E_{\gamma}$

E_{lib} appears as K.E of fragments (Y_1 & Y_2 & neutrons) also as radiant energy.

if Y_1 & Y_2 created at rest $\Rightarrow K.E_{Y_1} = K.E_{Y_2} = 0$

- Mechanism of nuclear fission: To realize (satisfy) the nuclear fission, the incident neutron must be thermal, that is, a slow neutron whose K.E is of order 0.02eV.

- Remarks: - The fission takes place by a neutron since it has no charge.
  - The energy liberated by the fission reaction is enormous
  - A nucleus is said fissile (fissionable) if it can undergo a fission reaction.
  - A nucleus is said to be fertile if it can generate a fissile nucleus by a certain reaction.

37 Nucleur Fusion:

- Definition: The fusion is a provoked nucleur reaction during which 2 light nuclei $^{A_1}_{Z_1}X_1$ & $^{A_2}_{Z_2}X_2$ unify (combine-emerge) to constitute a more heavy nucleus $^{A_3}_{Z_3}Y$
- Nucleur reaction: $^{A_1}_{Z_1}X_1 + ^{A_2}_{Z_2}X_2 \longrightarrow ^{A_3}_{Z_3}Y$ + particle(s) $+ ^0_0\gamma$
- Condition for fusion to take place:
  - X_1 & X_2 must have high kinetic energy to overcome the electrostatic repulsion
  - Temperature: 10^8 °K (Kelvin) {As temp ↑, K.E ↑}
- law of conservation of mass number:

$$(\Sigma A)_{before} = (\Sigma A)_{after}$$

- law of conservation of charge number:

$$(\Sigma Z)_{before} = (\Sigma Z)_{after}$$

- law of conservation of total energy:

$$\Sigma(mc^2 + K.E)_{before} = \Sigma(mc^2 + K.E)_{after} + E_{radiation}.$$

- Mass defect of reaction: $\Delta m = m_{before} - m_{after}$

$$= (m_{X_1} + m_{X_2}) - (m_Y + m_{particle(s)})$$

- Liberated energy: $E_{lib} = \Delta m c^2$

if $\Delta m > 0$ => reaction liberates energy: Exoergic
if $\Delta m < 0$ => reaction needs energy to take place (endoergic)

$$E_{needed} = |\Delta m| c^2$$

- form of E_{lib}:

$$E_{lib} = (K.E_{Y_3} + K.E_{particle}) - (K.E_{X_1} + K.E_{X_2}) + E_{radiation}$$

* The fussion reaction is non controlled while fission can be controlled

* $\underset{(Gy)}{Dose} = \dfrac{E_{total}\text{ recieved by body (J)}}{M\,(Kg)}$

* Physiological Equivalent Dose E.D

$$\underset{(Sv)}{E.D} = D \times Q.F$$

$$V = \frac{N_{rem} \times 10^{-2}}{N}$$